This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2023.3341060

Dynamic State Estimation for Improving Observation and Resiliency of Interconnected Power Systems

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***Abstract*—This paper investigates a particle filter (PF)-based fully-decentralised dynamic state estimation (DSE) method for interconnected multi-machine power systems. The PF-based observer is developed to dynamically estimate the states of the 7th-order dynamic model of synchronous machines that are either inaccessible and/or highly noisy. It is assumed that the proposed PF-based robust decentralized observer for a particular synchronous generating unit relies on typical output measurements available from phasor measurement units (PMUs) installed at its terminal. The performance of the presented observer is investigated using the benchmark model of the IEEE 68-bus system considering a detailed sub-transient representative model of synchronous machines with different excitation and control systems. The presented estimation framework works successfully and accurately under various transient events, such as load perturbation, faults, and changes in network topology, while accounting for different errors and sampling rates in measurements. The accuracy and robustness of the presented dynamic estimator in the case of Gaussian and non-Gaussian noisy measurements are verified. The paper also develops an approach-based PF to detect bad data and introduces a new metric based on the computation Cramér–Rao Low bound (CRLB) for evaluating the dynamic estimation performance. The introduced PF-based DSE improves the system resiliency by providing the system operator with the monitoring and observation capability of the system in a real-time manner to perform the proper corrective and protective actions in case of any events. The comparative study with other sophisticated dynamic state estimators confirms the brilliance, robustness, and superiority of the presented PF-based dynamic state estimation for multi-machine systems, and its practical and implementation feasibility.**

***Index Terms*—Decentralized estimation, state observer, state estimation, particle filter, phasor measurement unit, smart grid.**

I. Introduction

A. Motivation and Problem Statement

WITH the advancement of power systems, reliable and fast communication, online monitoring and observation, and effective control and coordination among various units of the system are imperative for its stable and secure operation [1]. These requirements are fulfilled using the remote terminal units (RTUs) and phasor measurement units (PMUs) at a fast sampling rate, providing real-time measurements at multiple

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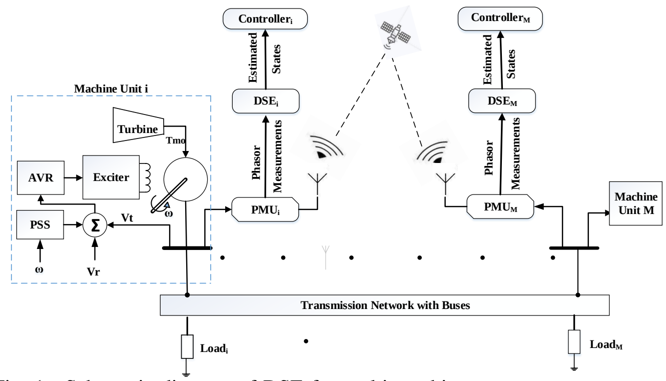

Fig. 1. Schematic diagram of DSE for multi-machines system

remote points synchronized using centralised global positioning system (GPS) and giving the command to control centers for anticipatory actions [2]. In these paradigms, redundant and imperfect measurements based on optical observations and/or radar signals may be contaminated signals and result in system measurement errors. Further, modern power systems are cyber-physical systems based on the coordination of physical systems with cyber communication and control layers. Any intentional or unintentional cyber malfunctions may mislead the system operator and a possibility of chaos in the complete system may arise. The aforementioned transitions of the power system enforce the need for real-time observation, monitoring, and capturing of the dynamic behavior of the system for secure and reliable operation. Therefore, dynamic state estimation (DSE) becomes crucial in modern digitized power systems.

The increasing complexity of power systems and the pressure faced by utilities in the deregulated energy market have led to a higher likelihood of faults occurring. Promptly clearing these faults and, specifically for synchronous machines, obtaining information about their internal states which can be affected by anomalies, faults, or data issues, becomes a crucial task to uphold the resilience of the power system. Although observation and monitoring of internal state variables of synchronous machines through the sensors are not feasible and cost-effective procedures for stability, resiliency, and security assessment of power systems. However, continuous state estimation during operating conditions using the available measurements makes the system reasonably observable. Capturing the dynamic behavior of states becomes important during network disruption not only enhance the observability but also the resiliency of the system if proper corrective measures are undertaken promptly.

Large-scale integration of distributed energy resources integrated with renewable energy sources(RES) on the generating side [1], as well as complicated loads and recent demand-response technologies on the demand side, such as electric vehicles and Internet of things devices, are exacerbating the dilemma [1]. As a result of the energy system transition, the system's dynamic properties have become more uncertain as compared to conventional ones. The presence of uncertainties in any part of the system results in the states of the system exhibiting stochastic behavior. As a result, the system's stochastic dynamics pose a challenge to legacy systems' typical deterministic model-based, real-time monitoring and control techniques [3]. State estimation provides a solution in case of inaccessible and contaminated states and overcomes the problems of redundant and imperfect measurements. In fact, fast, accurate, and robust estimation frameworks are needed for providing dynamic state information in real-time to the system operators to improve the monitoring, resilience, and observation capability of the system [4]. This is due to the fact that for better control and coordination of power systems, surveillance of systems can not rely only on the information received from SCADA for a steady-state model due to slow scan rates without timestamps. A need to estimate and update the states is required at an interval of a few milliseconds. Capturing of dynamic characteristics of states is a topic of interest, where its future research directions, challenges, and possible applications have been discussed in [5].

B. Literature Review

Dynamic state estimation deals with the recursive process of estimating and tracking the states of a system considering the system's dynamical model with the available measurements. As a model-based estimate procedure, extended Kalman filter (EKF) has found its viability to solve the nonlinear issue of DSE in power systems utilising Taylor series [6]. These studies have favored distributed DSE over centralised DSE in terms of decreasing communication delays and dispersing the computing burden to local control centers. However, the application of EKF is restricted for highly non-linear systems as i) the linearization procedure leads the estimator to diverge due to substantial inaccuracy in state distribution, resulting in limited performance; and ii) the computation of the Jacobian matrix is a difficult task. With the unscented Kalman filter (UKF), the linearization is skipped and statistical approximation is used to improve the EKF's first-order accuracy [7]. It also uses a nonlinear transformation to propagate the mean and covariance of the data and outperforms EKF estimates significantly. Further, studies on the estimation of synchronous machine states with EKF and UKF have shown that reliable estimation may be made by keeping data sampling interval and system simulation time step identical [8]. For better DSE, the particle filter (PF) has been suggested which is more accurate than EKF and UKF but requires more computing effort, however, the facility of high computing resources and high-fidelity data from PMUs with higher sampling rates can compensate for this limitation. In addition, the PF algorithm has the benefit of being able to handle nonlinear dynamics with both Gaussian and non-Gaussian noise errors that are included in system measurements [9].

The applications of particle filter as dynamic state estimator for power systems have been reported in literature[10]. It has been noted that resampling and careful selection of the importance density option is required when designing the PF for a particular application [11]. A DSE of a classical small-scale IEEE-3 machine 9-bus system has been tested with PF including a smoothing approach in [12]. PF with systematic resampling has been used to estimate the state variables of fourth-order multi-machine model [9] and later the work has been extended to include dual-estimation with the assumption of unavailability of measurement of field voltage [13]. In these studies, it has been observed that the accuracy in terms of root mean square error (RMSE) is reliant on the selection of the number of particles. Pointing to the limitations and implementation issues of PF as a state estimator in power system applications, [14] has proposed adaptive PF to enhance the robustness to offsets in the initial states and convergence ability, [15] has developed unscented particle filter (UPF) to overcome the problem of particle degeneration for doubly fed induction generator (DFIG) connected to a multi-area power system, [16] has proposed intelligent PF to overcome the concerns of sample impoverishment and degeneracy of the propagated samples during DSE of seventh order model of a machine system, and [17] has alleviated the problem of sample impoverishment using extended PF with iterative sampling in the single-machine infinite-bus system. Further, [18] proposed a technique to utilise PF with systematic resampling for the estimation of state variables of the fourth-order synchronous machine model. In addition, [16] and [18] have presented approaches to detect faulty data by linking it to gross measurement error in their significant works. The detection of bad/faulty data is one of the potential applications of DSE.

The real-time implementation of PF for DSE necessitates a computationally efficient approach, or one with high estimation accuracy and minimal computing time [19]. A study has been presented and claimed more computation efficiency with a reduced order model i.e. with lesser state variables [20]. Compared to UKF, the computational feasibility of PF implementation as a state estimator has been approved by varying the noise level [18] in a time frame of transient situations. Also, [6] has justified the computational feasibility of an estimation algorithm by implementing it on different benchmark systems. Irrespective of the size of the system, the iteration time of the DSE algorithm is within the measurement time range to implement the algorithm in real-time applications. The computational speed of any algorithm depends upon the language of the program as well as on the processor speed. With regards to the PF algorithm, [9] has shown that increasing the number of particles resulted in more accuracy but a slower speed of computation.

Measurements at terminals of active power, reactive power, current and voltage phasors are accessible through modern PMU. The estimation of dynamic states such as rotor angle and speed are necessary to ensure modern power systems are operated stably and securely, where they can be estimated using PMU measurements at each terminal if a decentralized technique is developed, assuming that synchronised voltage and current phasor measurements are available at every moment [5], [12]. With regards to the inclusion of noise in the measurement signal, Ref. [6] has assumed white Gaussian noise (WGN) of variance 10^{-8} - 10^{-10}. Furthermore, implementing UKF for DSE, [21] has extended his previous work by integrating UKF with interpolated discrete-time Fourier transform (DFT). This dual-stage estimation approach has relieved the estimation of the system dynamics from any synchronized measurements. It is noteworthy to consider the development of robust estimators [22] based on the investigation that PMU measurement errors deviate

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from the Gaussian probability distributions i.e. they can follow non-Gaussian distribution such as bimodal Gaussian mixture model, the Laplacian or the Cauchy distribution, and heavy-tailed distribution [23]. In such cases, PF can be a useful choice as a state estimator other than EKF and its advanced version [24], [25] because of its ability to capture non-Gaussian features by using different weights for different samples [10], [11]. The convergence of particle filter based estimation depends on how closely the estimated posterior distribution $f(p(\widehat{x}))$ approximates the true posterior $f(p(x))$. The convergence of PF in the presence of Gaussian & non-Gaussian noise is often discussed in terms of conditional expectation and mean square error [33], [34]. Although it is critical to examine the sudden alterations in load dynamics and system topology during dynamic estimation activity [26], which have been neglected in [7]. The robust DSE has been proposed during the situation of temporary loss of PMU communication link and abnormal state induced by impulse noise [27].

Various estimation methods have been tested on different models of synchronous generators with a variety of excitation systems considering different sizes of power systems in the last few years [17]. Likewise, various excitation system models have been suited for the examination of DSE and small-signal stability [28]. The algorithm is tested for state estimation in a synthetic Texas system in which low-level estimation-based DSE and subsequently, data-driven estimation using sparse selection is performed to deal with high dimensional systems. In the study of the estimation of dynamic states using EKF and UKF, the model of a synchronous machine connected to an infinite bus has been investigated and suggested to keep the data sampling time and simulation time equal to get the accurate estimation [29]. Further, using extended PF has been proposed in [30] for the estimation of dynamic states of a single-machine infinite-bus test system, neglecting sub-transient dynamics and saturation effects. A recent study report has investigated the utility benefits of DSE in a variety of control and protection applications along with comprehensive solutions for the stability and resiliency issues of modern power systems [31]. An observer-based nonlinear control strategy has been suggested in [32] which utilises major estimated states as feedback for the stability enhancement of a multi-machine power system.

C. Industry Issues and Research gaps

The aforementioned survey presents the research gaps and verifies the need for research on DSE that employs realistic sample rates, considers practical transient situations, and takes alternative model architectures for filtering and simulation into account, for widening the application of state estimation techniques in real-world power systems. There is an obvious need to investigate a suitable dynamic state estimator to be used in modern power systems that are going under energy transition due to energy decarbonization plans around the globe which impose considering a higher order of the dynamic model of synchronous machines and other active elements in the grid in order to thoroughly investigate stability, resiliency, and security issues and propose advanced control and protection schemes. Due to energy decarbonization, the traditional coal-fired power plants are being retired and replaced with either renewable energy sources including hydro-power plants and solar and wind energy sources. Additionally, great projects around the globe are being initiated to use hydro-pumped power plants for empowering energy decarbonization, especially in China, the USA, and Australia. Such power plants utilise Salient-pole type or projected pole type rotor synchronous machines where their dynamic models are different from others due to the differences between d-axis and q-axis transient/sub-transient reactance. For instance, the 6^{th} order dynamic model of the ordinary synchronous machine becomes 7^{th} order dynamic model in machines used for aforementioned power plants. The 7^{th} order model can not be easily reduced to a lower order model; for instance, the used 4^{th} order model for synchronous machines adopted for DSE in literature can not be accurately utilised for synchronous machines with a specific design of their rotors mentioned above. None of the previous studies in the literature has investigated the capability of using either PF or UKF for DSE in such types of synchronous machines with higher dynamic order and more complex dynamics due to their rotor special design, which is a real need for industry around the globe where this research is designed based on an industry project highlighted such needs. Similarly, there is a need to investigate the feasibility of PF for DSE for practical implementation with any of the following possibilities: (i) any type of probabilistic noise mix measurements due to restructured power systems, (ii) varying sampling rates of different types of PMUs, and (iii) computational speed of filter which should be less than PMU observation between two successive readings. Similarly, there is a need to investigate the capability of PF for DSE in the case of non-Gaussian noise, especially those noises that might affect dynamic estimation such as Laplacian noise. Moreover, an investigation of the impact of different types of sampling techniques is needed for the industry application of DSE. The research work in this paper is dedicated to addressing the aforementioned research gaps and industry challenges and issues.

D. Contribution

This paper addresses research gaps (discussed in I.B) and industrial challenges (Discussed in I.C) associated with DSE in modern power systems and confirms the implementation and application feasibility of DSE in power systems. Our work develops PF-based DSE for the 7^{th} dynamic order model of synchronous machines and extensively compared it with the introduced UKF-based DSE. The proposed PF-based DSE method in this paper is developed to be a fully decentralized one, providing new advantages to DSE applications in power systems. Among the brilliant features of the introduced fully decentralized PF-based DSE, the DSE applied to each synchronous machine is totally independent of other machines' state estimation processes, thus errors remain localised and simpler to identify, and the process is more secure and reliable if compared to central estimate techniques. The developed DSE in this paper avoids the loss of data transfer and delays and solves the constraint of sampling speed of the signal due to the communication network capacity. This paper introduces a robust filter against noises and mathematically proves the capability of the proposed DSE in handling industry issues and technical challenges associated with both Gaussian and non-Gaussian noises. Furthermore, the Laplacian noise is modeled and considered, where the results confirm the robustness of the proposed PF-based DSE. This paper also investigates the impact of different types of resampling techniques, i.e. systematic, multinomial, and stratified resampling techniques, on the performance of DSE. This paper also introduces a CRLB metric analytically confirming the convergence of the proposed DSE method and the effectiveness of the proposed DSE in dealing with non-Gaussian noise distributions. Moreover, the paper develops a procedure to detect bad data and confirm the capability of the proposed DSE application for renewable energy resources-dominated power systems. Overall, the performance of the proposed dynamic state estimation technique is superior in comparison with other estimation approaches. The major contributions of this paper are summarised as follows:

- Development of a new realistic PF-based DSE specifically for a detailed model of Synchronous Machines considering sub-transient 7th order dynamic model and the machine's different controllers.
- The Development of a realistic DSE based on a full decentralisation concept capable of avoiding the impact of the loss of data and communication network capacity constraints on the estimation process.
- A new metric based on the computation Cramér–Rao Low bound (CRLB) for evaluating the performance of the PF-based DSE estimator.
- Development an advanced PF-based DSE enjoys important features including i) computational time far less than a PMU sampling interval; ii) excellent tracking capability, even when dealing with noisy data characterized by probability distributions of various shapes; (iii) effectively capturing of system dynamics at different sampling rates.

E. Organization

The paper continues with Section II which introduces the assumptions adopted in this research, and Section III presents the dynamical model of a multi-machine bus system followed by Section IV to demonstrate the implementation of particle filter. The procedure of PF-based DSE is introduced in Section V. The system under investigation is introduced in Section VI. The simulation results and related findings are presented in Section VII and performance analysis and main findings are given in Section VIII, while Section IX concludes.

II. Assumptions & Notations

A. Assumptions and Consideration of Practical Issues

This paper assumes, without loss of generality, that the information regarding the machine dynamic model, PMU local measurements and noise covariance is available. Refer to Fig. 1, it is considered that a PMU is installed in each generating unit terminal for measuring the voltage and current phasors. Assuming that the global positioning system (GPS) which is a satellite-based radio navigation system, is used to synchronise all of the PMUs in the power system with one absolute time reference. Also, it is shown that GPS receivers are installed that receives the signal and resolve it in three-dimensional time coordinates. It considers practical PMUs that are capable of retaining sequence of signals' information at sampling rates with a measurement accuracy (of $\approx \pm 0.1$ mrad for phasors). To take the practical issues into account, the PMU measurement is regarded to be a valid signal with measurement noise which may be additive white Gaussian noise (WGN) with zero mean and standard deviation equal to the PMU's accuracy for that particular measurement. The Gaussian distribution is justified on the basis of the Central Limit theorem, assuming the availability of large samples. Although the proposed method is a fully decentralized DSE technique, a detailed topology of the network is considered in the dynamic study. With not losing the generality, the sluggish speed-governor dynamics is neglected and treating the mechanical torque as constant during the sub-transient period based on the fact that the governor takes more time to start acting due to its and turbine's dynamic constraints and delays. To provide flexibility in adopting the model and its inputs and outputs, the study is designed such that with the available local measurements of a machine's voltage phasor and current phasor, one can be considered as input and the other as output in order to separate the estimation of one machine unit from the estimation of other connected machines, resulting in fully decentralized dynamic estimation technique.

B. Terminologies & Variables definition

The paper uses the following nomenclatures and terminologies, n, m are the number of states, and outputs accordingly, i is the generator index, E_{fd} is the field excitation voltage (p.u.), D_i is the machine damping (p.u.), H_i is the rotating inertia (s), T_0 is the system sampling time (s), V, I are the voltage and current at the machine terminal, θ, ϕ are phasors angle of voltage and current (p.u.), I_d, I_q are d-axis and q-axis components of the stator current (p.u.), V_d, V_q are d-axis and q-axis component of the stator voltage (p.u.), T_m, T_e are mechanical and electric air-gap torques (p.u.); ω_j, ω_o are rotor speed for generator j (p.u.) and its base value (rad/s), δ_j is the rotor angle of synchronous machine j (rad); X_d, X'_d, X'_q are synchronous and transient reactances of d and q axes (p.u.),X_ls is the armature leakage reactance (p.u.), T'_{d0}, T'_{q0} are transient time constants of d and q-axis (s), T''_{d0}, T''_{q0} are sub-transient time constants of d and q axis (s), E'_q, E'_d are d and q axis transient emf (p.u.), ψ_{2qi}, ψ_{1di} are d and q axis sub-transient emfs (p.u.).

Fig. 1 depicts the layout of decentralised state estimation of the multi-machine power system. It shows the decentralized concept adopted for estimating the dynamic states in each synchronous machine separately from other machines. The sub-transient model of a synchronous machine is described in Section III, while the associated dynamic state estimator is introduced in Section IV.

III. POWER SYSTEM MODEL

The general state space model of a system in discrete form can be represented as,

$$\left.\begin{aligned} x(k) &= f(x(k-1), u(k-1)) + \xi(k) \\ z(k) &= h(x(k), u(k)) + \nu(k) \end{aligned}\right\} \quad (1)$$

where the system's current state vector, x at step k is determined by the system's prior state, input, and system process noise i.e. x_{k-1}, u_{k-1} and $\xi(k)$, respectively. The measurement process, $z(k)$ has a nonlinear function, $h(.)$, of states and inputs, in addition to the measurement noise, $\nu(k)$. The $z(k)$ is known as the measurement (observation) vector. Also, $\xi(k)$ and $\nu(k)$ are assumed to have independent and identical distribution (iid). It is assumed that the initial condition $x(0)$ is known or estimated and that the noise terms are independent random variables with probability distribution function (pdf). Using the system and measurement model in (1), a dynamic estimator is required to

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estimate the next state recursively. The objective of the observer is to estimate the state, $\widehat{x}(k)$ given $\widehat{x}(k-1), u(k-1)$, and the observation vector $z(k)$.

The primary requirement of DSE for a synchronous machine is to obtain the model of system dynamics. For this, a comprehensive 7^{th}-order sub-transient model of a synchronous machine is adopted. Likewise, an interconnected multi-machine system, i.e. a 16-machine, 68-bus test system, is considered to demonstrate and investigate the proposed DSE approach. The machines have transient and sub-transient dynamics, as well as several types of dynamic models for excitation such as DC exciter and static exciter. A schematic diagram of interconnected 5 areas, IEEE-68 bus, 16 machines system is depicted in Fig. 3, where its DSE approach-based single line diagram is shown in Fig. 1. The diagram represents different components like synchronous machines, load, and network, where all have connections through different buses. The power is exchanged among these areas through double-circuit tie-lines such as power areas NETS and NYPS are connected through bus 53-54 and two more buses as shown in Fig. 3. The discrete form of differential-algebraic equation (DAE) of the i^{th} synchronous machine is represented as

$$\delta_i(k+1) = \delta_i(k) + T_o\omega_b(\omega_i(k) - 1) \tag{2}$$

$$\omega_i(k+1) = \omega_i(k) + \frac{T_o}{2*H_i}(T_{mi} - T_{ei}(k) - D_i(\omega_i(k)-1)) \tag{3}$$

$$\begin{aligned} E'_{qi}(k+1) &= E'_{qi}(k) + \frac{T_o}{T'_{d0i}}[E_{fdi}(k) - E'qi(k) + \\ &(X_{di} - X'_{di})[K_{d1i}I_{di}(k) + K_{d2i}\frac{\psi_{1di}(k) - E'_{qi}(k)}{X'_{di} - X_{li}}] \end{aligned} \tag{4}$$

$$\begin{aligned} E'_{di}(k+1) &= E'_{di}(k) + \frac{T_o}{T'_{q0i}}[E'di(k) + (X_{qi} - X'_{qi}) \\ &K_{q1i}I_{qi}(k) + K_{q2i}\frac{\psi_{2qi}(k) + E'_{di}(k)}{X'_{qi} - X_{li}}] \end{aligned} \tag{5}$$

$$\begin{aligned} \psi_{2qi}(k+1) &= \psi_{2qi}(k) + \frac{T_o}{T''_{q0i}}[I_{qi}(k)(X'_{qi} - X_{li} \\ &- E'_{di}(k) - \psi_{2qi}(k) \end{aligned} \tag{6}$$

$$\begin{aligned} \psi_{1di}(k+1) &= \psi_{1di}(k) + \frac{T_o}{T''_{d0i}}[I_{di}(k)(X'_{di} - X_{li} \\ &- E'_{qi}(k) - \psi_{1di}(k)] \end{aligned} \tag{7}$$

$$\begin{aligned} E'_{dci}(k+1) &= E'_{dci}(k) + \frac{T_o}{T_{ci}}(X''_{di} - X''_{qi})I_{qi}(k) \\ &- E'_{dci}(k)) \end{aligned} \tag{8}$$

where, $K_{d1} = \frac{X''_d - X_l}{X'_d - X_l}$; $K_{d2} = \frac{X'_d - X''_d}{X'_d - X_l}$; $K_{q1} = \frac{X''_q - X_l}{X'_q - X_l}$ and $K_{q2} = \frac{X'_q - X''_q}{X'_q - X_l}$.

In the system under consideration, three types of excitation systems are considered as indicated in Fig. 3. As aforementioned, slow dynamics are not taken into account and mechanical torques are assumed to be constant during the sub-transient period. The configuration of different machine units is as: i) machines 1-8 and machines 10-12 are with IEEE-DC 4B type AVR without PSS, therefore, their state vector is $X_i = \begin{bmatrix}\delta_i & \omega_i & E'_{qi} & E'_{di} & \psi_{2qi} & \psi_{1di} & V_{ri} & V_{di} & E_{fdi}\end{bmatrix}^T$ where i= 1,2, ...8 & 10..12; ii) machine 9 is assumed to have an IEEE-ST1A type as an AVR and a PSS, resulting in that $X_i = \begin{bmatrix}\delta_i & \omega_i & E'_{qi} & E'_{di} & \psi_{2qi} & \psi_{1di} & E_{fdi}, P_{s1i}, P_{s2i}, P_{s3i}\end{bmatrix}^T$ where i=9; and iii) machines 13-16 are with manual excitation without PSS, therefore, their state vector is $X_i = \begin{bmatrix}\delta_i & \omega_i & E'_{qi} & E'_{di} & \psi_{2qi} & \psi_{1di} & E_{fdi}\end{bmatrix}^T$ where $i = 13, 14, ..., 16$.

The following section presents a decentralized estimation framework that employs an iterative estimation procedure for each type of synchronous machine.

IV. Proposed PF-based Dynamic State Observer

In this section, a Bayesian estimation methodology is utilized to develop a particle filter for estimating the dynamic states described earlier. This estimation process relies on local data and measurements obtained from PMUs installed at the generators' terminals. The evolution of the state vector, $x(k)$ and the measurement vector, $z(k)$ follows the representation provided in Equation 1. The algorithm leverages these measurements to update and refine its understanding of the state at time $t = k$. A probability density function $p(x_k|z_k)$ is formulated, assuming the availability of a prior PDF as $p(x_0|z_0)$, even in the absence of any measurements. This PDF, $p(x_k|z_k)$, is constructed recursively through two steps: prediction and correction.

For nonlinear systems with non-Gaussian distributions, an optimal Bayesian filtering approach can be developed without explicitly involving a model. Instead of using a system model, the posterior density function is represented using a set of random samples, denoted as $(x_{0:k}^{[j]}, \omega_k^{[j]})^{N_s}$. This approach serves as a recursive state observer, continuously updating its belief about the state particle $x_{0:k}^{[j]}$ based on the incoming sequence of measurements, denoted as $z_{0:k} = z_1, z_2, ..., z_k$. These particles can take on various values, representing different possible realizations of a state sequence. The number of particles is denoted as N_s, and each particle $x_{0:k}^{[j]}$ represents a potential state sequence realization. A weight $\omega_k^{[j]}$ is assigned to each particle to indicate its relative importance among all N_s particles. It is crucial to note that the sum of all particle weights $\omega_k^{[j]}$ from $j = 1$ to N_s is equal to 1.

The discrete weighted approximation of the true posterior $p(x_{0:k}|z_{1:k})$ can be expressed as:

$$p(x_{0:k}|z_{1:k}) \approx \sum_{j=1}^{N_s} \omega_k^{[j]}\delta(x_{0:k} - x_{0:k}^{[j]}) \tag{9}$$

Here, the Dirac delta function is denoted as $\delta(\cdot)$. The weights are selected by the principle of importance sampling with the assumption:

$$p(x) \sim \pi(x); \text{and } x_k^{[j]} = q(x_k|x_{k-1}^{[j]}; z_k), j = 1, 2, ...N_s \tag{10}$$

Since it might be difficult to directly sample from $p(x)$, the density function $\pi(x)$ can be formulated, and samples can be easily drawn from the importance density $q(x)$.

The computation of the importance weights is as follows:

$$w_k^{[j]} \propto \frac{target(x_k^{[j]})}{proposal(x_k^{[j]})} = \frac{p(x_{0:k}^{[j]}|z_{0:k})}{q(x_{0:k}^{[j]}|z_{0:k})} \tag{11}$$

In recursive filtering, the primary focus is typically on the state at time step k, denoted as $p(x_k|z_{1:k})$, rather than the entire state sequence up to time k, $p(x_{0:k}|z_{1:k})$. Therefore, (11) can be reformulated to prioritize the estimation of the specific state at time k.

$$w_k^{[j]} = w_{k-1}^{[j]} \frac{p(z_k|x_k^{[j]})p(x_k|x_{k-1})}{q(x_k|x_{k-1}, z_k)} \tag{12}$$

Without delving into additional derivations, the resulting filtered posterior density is expressed as:

$$p(x_k|z_{1:k}) \approx \sum_{j=1}^{N} \omega_k^{[j]} \delta(x_k - x_k^{[j]}) \tag{13}$$

The weights are normalized such that the weights of all particles will be proportional to their likelihoods relative to each other and the required condition is as below.

$$\sum_{j=1}^{N_s} \omega_k^{[j]} = 1 \tag{14}$$

Additionally, to address the problem of degeneracy in the PF algorithm, the following measure is assessed:

$$N_{eff} = \frac{1}{1 + \text{var}(\omega_k^{[j]})} \approx \frac{1}{\sum_{j=1}^{N_s} (\omega_k^{[j]})^2} \tag{15}$$

Where, N_{eff} represents the required effective sample size, and $\omega_k^{*[j]}$ denotes the true weight of the j^{th} particle. Additionally, it is important to ensure that $N_{eff} \leq N_s$ to avoid the degeneracy issue in particle filters, which can lead to divergence in estimation. To mitigate this problem, one approach is to set a very large value for N_s, but this may be constrained by slow computational speed and increased efforts. A solution to address this challenge is to employ the resampling step, which helps in alleviating the degeneracy issue and ensures more accurate estimation without requiring an excessively large value for N_s. The estimate state is given below.

$$\widehat{x} = \sum_{j=1}^{N_s} (\omega^{[j]} x^{[j]} \tag{16}$$

This study utilized various sampling techniques to assess the performance of each method individually during DSE of the generator states.

Before conducting simulation experimentation, the upcoming section demonstrates the procedure of implementation of observer for the execution of DSE of the synchronous machine in a multi-machine set-up.

V. Procedure Description of DSE of Synchronous Machine with Proposed Filtering Framework

The estimation of the internal states of a synchronous machine, as defined in Equations (2) to (8), requires the utilization of current measurements (I_g) with their d-axis and q-axis components (I_d and I_q) using the following expressions:

$$I_{gi}(k) = (I_{qi}(k) + I_{di}(k))e^{j\phi_i(k)} = Y_i(k)V(k) \tag{17}$$

where, $i = 1, 2, ...M$; $Y_i(k)$ is the i^{th} component of admittance of bus in p.u., $Y(k)$; In multi-machine system, disturbance at any part may affect all the voltages and currents of buses those ultimately affect the sub-transient and transient dynamics of machines. During time-domain simulations, a sequence of measurements of individual machine are available which upon any disturbance keep on changing. For practical systems, these are local PMU measurements available at generator terminal buses which are independent of other generators and the system-wide measurements. From Fig. 2, it is inferred that coordinating control with AVR and PSS keep the states of machine in stable whenever they perturbs due to any disturbance occurs. The simulation environment is formulated to create various transient conditions (at different instants) to introduce perturbation in the states of a machine. The simulink model has been altered to allow the application of the proposed filter to any machine within the system. This modification enables the tracking of the state vector trajectory of a specific machine, allowing for observation and analysis of its behavior under dynamic changes. The purpose of utilizing an observer is to estimate the state vector, which comprises seven internal states representing its dynamics, i.e. $\widehat{X} = \begin{bmatrix} \widehat{\delta} & \widehat{\omega} & \widehat{E'_q} & \widehat{E'_d} & \widehat{\psi_{2q}} & \widehat{\psi_{1d}} & E_{fd} \end{bmatrix}^T$. PF is implemented and simulated in a decentralised manner for the estimation of state variables of each machine.

For this paper, a self-presentation block diagram of PF is depicted in Fig. 2 according to the dynamical model (refer Eq. (2)-(8)) which will be obtained in the Simulink model using state transition function prior to implementation of discrete-time PF algorithm. In addition to the appropriate state transition function, the measurement likelihood function is also created for the system. This function computes the likelihood of particles from the noisy measurements assuming a Gaussian measurement noise distribution. The PF is configured according to state functions and initial conditions of the states. It propagates uncertainity by simulating and determining the posterior estimates through importance sampling.

In what follows, the performance of the proposed PF-based DSE is investigated on the aforementioned IEEE 68-bus system by several scenarios in order to verify its superiority and implementation feasibility. These scenarios are fabricated to replicate the real power system transients/faults situations. In such cases, knowing the real-time state information of a synchronous machine is critical in order to detect instabilities and compute suitable control measures. In the present system, the field winding of synchronous generator is either manually excited or through AVR where, this controller take in charge of keeping voltage constant at generator terminals. In simulations, initialization is accomplished by assigning initial conditions in the block, followed by a prediction step that employs the most recent state to predict the next state based on the state transition model. Furthermore, the correction rectifies the state estimate using the current measurement. We have also conducted simulations using UKF as state estimator for the same test model so to imitate inaccurate PMU measurements, white Gaussian noise (WGN), $\nu \sim N(0, \sigma_\nu)$ is introduced to the sampled values of I_g as measurement noise. For simulations, the initial values such as the initial process noise covariance matrix and measurement noise covariance matrix (no need to set in case of PF), are all the same, except for the initial state vector, which has varying initial values of δ and ω for different machines. We have introduced process noise in each machine model. The estimated states from the associated machine will be simulated as a result. Considering $\mathbb{E}$ as the expectation operator, the form of the state space model given in (1) for implementation of the PF algorithm

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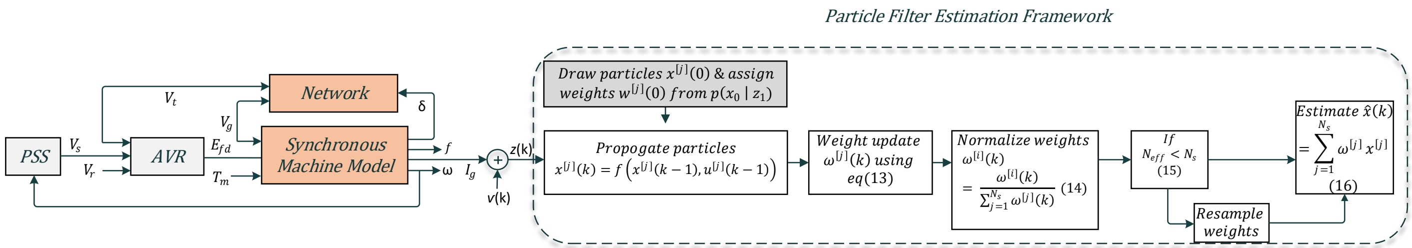

Fig. 2. Procedure description of DSE of ith Synchronous machine of a multi-machine model.

can be written as:

$$\left.\begin{aligned} &\dot{x} = f(x,u) + \xi \\ &y = h(x,u) + \nu \\ &\mathbb{E}[\xi \;\; \xi^T] = Q_w; \quad \mathbb{E}[\nu \;\; \nu^T] = R_v \\ &X = [\Delta\delta \;\; \omega \;\; \Delta P_r \;\; \Delta E'_{qi} \;\; \Delta E'_d \;\; \Delta\psi_{2q} \;\; \Delta\psi_{1d}] \\ &u = [E_{fd} \;\; T_{m0} \;\; V_q \;\; V_d \;\; \omega]; \quad z = [I_q \;\; I_d] \end{aligned}\right\} \tag{18}$$

It is assumed that each machine includes a PMU that facilitates measuring the different phasors associated with that machine, especially the voltage and current phasors and each measurement sample arrives as input to filter at a sequence of 10 ms. In order to produce online estimation, every iteration of the algorithm should produce result within 10 ms. State transition function (StateTransitionFcn) is a function responsible for computing the particles (state hypotheses) for the upcoming time step, given the state vector at the current time step. On the other hand, the Measurement Likelihood Function (MeasurementLikelihoodFcn) is a function responsible for determining the probability or likelihood of each particle based on the sensor measurements. The detailed procedure is illustrated with the help of pictorical representation as shown in Fig. 2.

VI. System Under Consideration

To assess the performance of the proposed PF-based DSE technique, a large-scale IEEE standard power system model is considered. This model encompasses both the generator dynamics and load bus power flow data, integrated into a unified framework. Thus, the simulation study is carried out using an IEEE 68 bus and 16 machine interconnected 5 areas model, which is also known as the interconnected New England test system (NETS) and New York power system (NYPS) (refer Fig. 3 & Fig. 2). The model includes power system stabilisers and automated voltage regulators, with dynamic states acting as feedback signals to maintain power system stability. The simulation data was created using a complicated model including sub-transient dynamics, saturation effect, exciters, and PSSs to simulate the fact that the accessible model is a simplified approximate setup of the real system. In this study, the value of base power is set to be 100 MVA, and the system frequency is 60 Hz. Three different types of machines with different configurations of AVR and PSS have been selected for results realization namely machine 3 from NETS i.e. M/c 3 (Type 1), machine 9 from NETS i.e. M/c 9 (Type 2), and machine 13 i.e. M/c 13 from NYPS (Type 3). These machines have their own set of parameters and configurations, such as M/c 13 has the highest mechanical torque (T_m) among other machines. Being a decentralised estimation approach, the

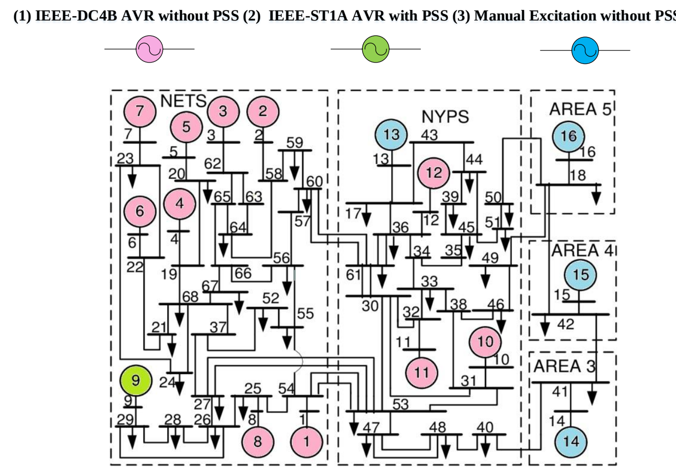

Fig. 3. Line diagram of IEEE-68 bus, 16 machines system.

dynamic state estimator algorithm implemented to any machine will work independently from all other machines integrated with the grid.

VII. Simulation Environment, Study and Findings

A computing system, with specifications: 3.4 GHz core, i7 CPU, 8GB RAM, and MATLAB software (version 2020b), is used for the simulation study. The performance of the particle filter as a dynamic state estimator is validated by creating a simulation model for a multi-machine system and then running various simulation scenarios that simulate transients that may occur in real-world power systems. Since the majority of PMUs can sample 30 measurements per second, according to a NERC report. Further, considering the availability of more advanced sensors capable of sampling at a much higher rate. We have chosen to use a 0.01 sampling rate in this proposed work.

A. Simulations and Result Analysis

The efficacy of the proposed filtering algorithm is validated by introducing the following dynamics simulation scenarios in the above-discussed power system model and a simulation study of 15 seconds is executed.

1) Dynamic Situation #1: A simulation study of 15 seconds in which a transient is created at the time, t = 3.0 s by providing a step of 2% in V_{ref} and at t = 12.0 s, further a step of -2% is provided to the same V_{ref}.

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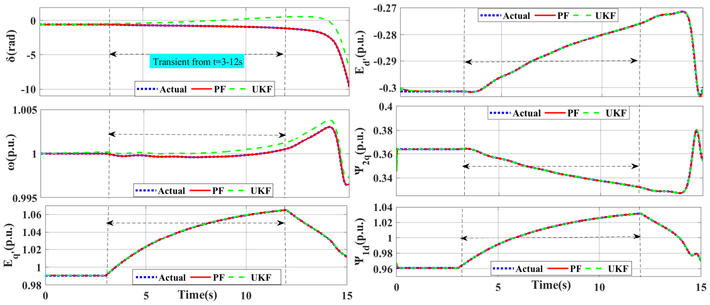

Fig. 4. Simulation results of Dynamic Scenario #1 for M/c 13: States estimation with Cov.10^{-8}

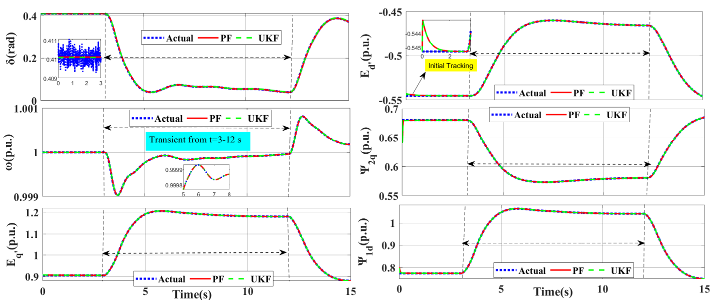

Fig. 5. Simulation results of Dynamic Scenario #1 for M/c 3: States estimation with Cov.10^{-8}

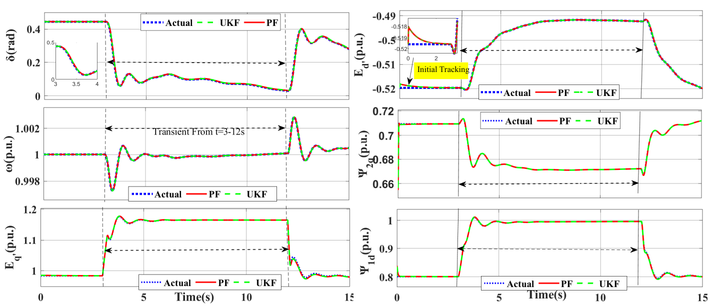

Fig. 6. Simulation results of Dynamic Scenario #1 for M/c 9: States estimation with Cov.10^{-8}

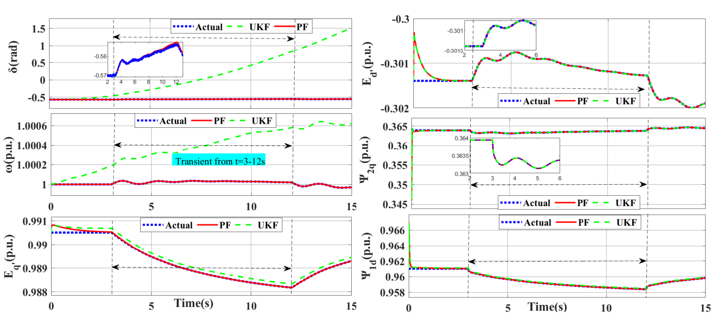

Fig. 7. Simulation results of Dynamic Scenario #2 for M/c 13: States estimation with Cov.10^{-8}

2) Dynamic Situation #2: Connection of a 50 MVAr shunt reactor to the bus linked with the test machine at t = 3.0 s, followed by the removal of this shunt reactor at t = 12.0 s. Both dynamic situations, #1 and #2 are created with a

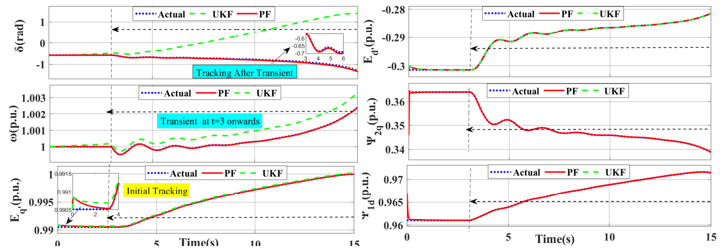

Fig. 8. Simulation results of Dynamic Scenario #3 for M/c 13: States estimation with Cov.10^{-8}

fixed value of mechanical torque (T_m) assigned for each machine.

3) Dynamic Situation #3: A transient is created by lowering the respective T_m by 5% of the chosen machine starting at time, t = 3.0 s from its initial value.

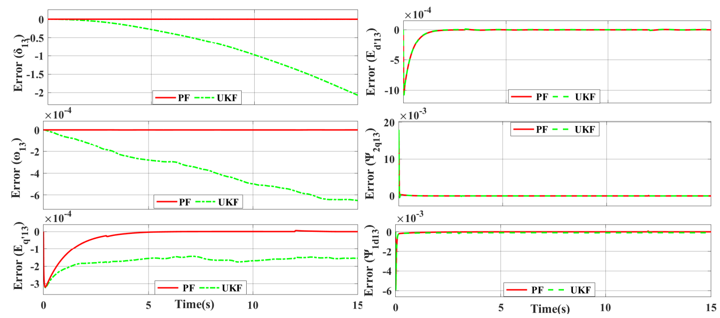

Fig. 9. Simulation results of Dynamic Scenario #1 for M/c 13: Estimation Error of 6 states with Cov.10^{-8} using UKF and PF

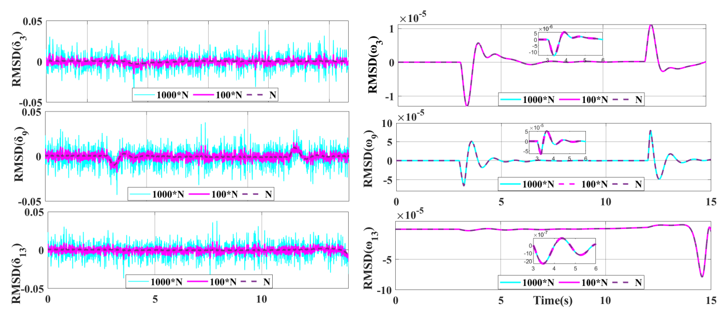

Fig. 10. Simulation results of Dynamic Scenario #1 for M/c 3, 9. and 13: Effect of changing Noise(N) on RMSD for State 1 and state 2.

The aforementioned dynamic situations are included during the estimation of the 3 machines with different PSS and excitation models, i.e M/c 13, M/c 9, and M/c 3. The estimation is performed using the proposed PF-based DSE technique and compared to an advanced DSE technique, i.e. UKF. After executing several simulation experiments, comprehensive findings are displayed in Fig. 4- Fig. 10 and Table I- Table II. The tables reflect root mean square deviations (RMSD) between the actual theoretical value and the estimated value of state variables at different machines for different conditions and dynamic situations. The expression for RMSD (between actual state (x_{ij}) and estimated state ($\widehat{x}$)) is $\sqrt{\frac{1}{N}(\sum_{j=1}^{N}(x_{ij}-\widehat{x})^2)}$; where,

$i = 1, 2, ..., n$. n and N denote the number of states and samples, respectively.
Assuming sufficiently large random samples, it is considered that the measurement and process noise have Gaussian distribution (follow the Central Limit theorem) and under this ideal case, the comparative study of UKF and proposed PF-based observer for generator system has been conducted. In addition, to demonstrate the robustness of PF as DSE to simulate the system with non-Gaussian measurement and process noise with adequate dimensions, a signal representing the noise of Laplacian distribution is included. Unlike the normal distribution, the Laplace distribution (LD) has the sharpest peak and is defined with location parameter (θ) and scale parameter (s). Its probability distribution on $(-\infty, \infty)$ is defined by density function:

$$f(x; \theta, s) = \frac{1}{2s} \exp^{\frac{-|x-\theta|}{s}} \tag{19}$$

where, $-\infty < x > \infty$; $\theta \in (-\infty, \infty)$ and $s > 0$. In the simulation, a function block of random number generator with LD is used and a signal is added as process and measurement noise using the function: function $y = laprnd()$; where, $y = 0 - (1/\sqrt{2} * sign(rand - 0.5) * log(1 - 2 * abs(rand - 0.5))$. Here, mean = 0 and scale $=1/\sqrt{2}$.

The performance of the estimates is evaluated from these

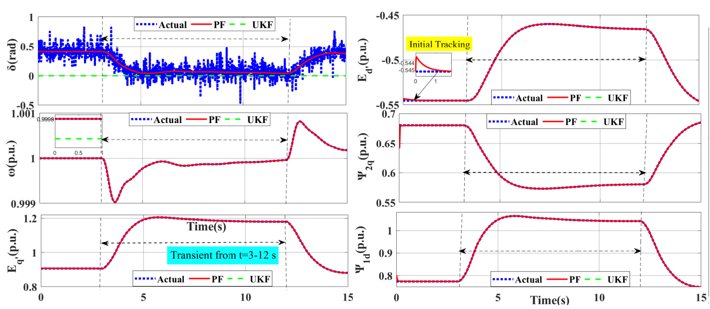

Fig. 11. Simulation results of Dynamic Scenario #1 for M/c 3: States estimation with Laplacian noise using UKF and PF.

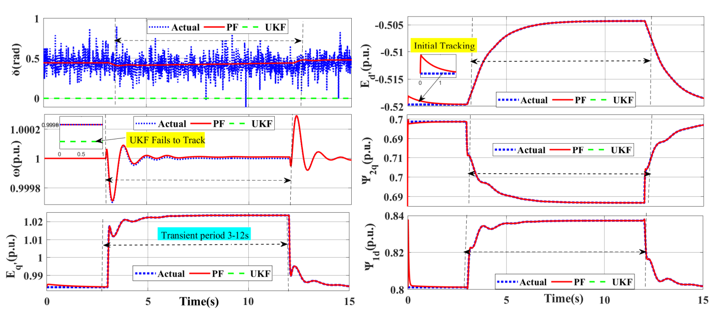

Fig. 12. Simulation results of Dynamic Scenario #2 for M/c 9: States estimation with Laplacian noise using UKF and PF.

simulation tests, the results are analysed to validate the simplicity, correctness, speed, and practical feasibility of the proposed estimator of PF-based DSE for the introduced complex and comprehensive dynamically-modeled multi-machine power system. The main points inferred from the simulation study are discussed below.

- Knowledge of initial conditions is critical for accurate estimation otherwise, it may lead to high estimation errors. It is different for each machine and must be set precisely for all states to achieve perfect estimation results. The same initial conditions are used for UKF and PF implementations during the simulation study.
- While utilizing other advanced DSE techniques such as UKF as a state estimator, knowledge of the noise covariance (both process and measurement) must be taken into account in addition to the initial conditions of the state transition matrix and measurement function for a sub-optimal estimate while in case of PF, no such knowledge of noise covariance is required.
- In an interconnected multi-machine system, a change in the dynamics of any of the machine unit affects the dynamics of other machines. Additionally, the state behaviour of each generating machine is suffered by the transients in any part of the system. Regardless, all states of an individual generating machine are well estimated using PF algorithm with excellent tracking in all cases. This is due to the decentralised approach used for estimating each machine's state variables based on information and measurements of that specific machine.
- For all simulations, a minor estimation error in any of the states, particularly state 2, X_2, i.e. ω, leads to a large estimation error in rotor angle, δ. Referring to Tables I and II, the range of estimation error(RMSD) for all states is 10^{-5}, with the exception of δ, which is significant. The comparative study verifies the superiority of PF-based DSE which provides a higher accuracy and makes its implementation in real-world power systems feasible.
- The level of noise affects the accuracy of the estimation. As the noise level increases, the estimation error increases (refer Table I but the estimation does not lead to divergence even the base noise covariance of process noise ($Q_w = 10^{-8}$) and measurement noise ($R_w = 10^{-8}$) is increased to higher levels. This is reflected in Fig. 10 as estimation error for 2 states (δ & ω) of M/c 3, 9, and 13 under three dynamic situation #1, #2 and #3. Further, the efficacy of the estimation algorithm is investigated by varying the noise level (covariance), and it is found that estimation is accurate up to covariance 10^{-5} but afterward has very high RMSD.
- Based on a simulation study utilising the UKF and PF estimators on the same model with the same initial conditions, it was discovered that estimation error using UKF or all states is somewhat higher than estimation using PF. This difference is minor for M/c 3 and M/c 9, so we have displayed only scenario #1 for these machines due to space constraints. But UKF estimates for M/c 13 fails to converge the actual state of δ after transients (refer to Figs. 4, 7, and 8). Initially, UKF tracked the actual δ, but it failed to track when a transient occurs at $t = 3s$ (dynamic Scenario #1, #2 and #3) where UKF is unable to capture the actual dynamics of δ and ω during these dynamic scenarios. For each machine, PF outperforms UKF specially for M/c 13 (refer Figs. 7, 8 and 9). This validates the robustness of the PF algorithm to track the system transient behavior at each instant across machines as compared to the UKF algorithm. The associated estimation error of both UKF and PF can be

inferred from Table II.

- The computing time for the aforementioned test scenarios is observed during the simulations. One iteration of the PF algorithm takes < 3ms for each case, which is less than the measurement time (10ms).
- Dynamic simulations conducted on the different machine systems with the inclusion of non-Gaussian noise (Laplacian noise) are shown in Fig. 11 and Fig. 12 indicating the robustness of the proposed PF estimation approach where UKF fails to track the true states.
- The performance of the proposed PF as a dynamic estimator is further investigated on the same system with different types of resampling techniques (such as systematic, multinomial, and stratified resampling) and variations in the number of particles (1-1000) of proposed PF. It is observed during simulations (both with Gaussian and non-Gaussian noise) that there is no significant change in the RMSD of all states of various types of machines during transient scenarios considering different resamplings and particles. However, the time of simulation execution varies depending on the type of resampling and the number of particles applied. The results are summarized and given in Table III. The simulation execution time increases as the number of particles increases, and in relation to the nature of the noise structure, the execution time of the present DSE algorithm with non-Gaussian noise is more than with Gaussian noise.
- The simulation study is conducted using a sampling rate of 10 ms and 20 ms. Accordingly, the PF and UKF algorithms are configured to perform the DSE for different machines at different cases of abrupt state transients. With respect to the change in the sample time, the estimators work well.

B. Sensitivity Analysis and the Impact of High Integration of IBRs

In this subsection, the robustness of the developed dynamic estimator is assessed against variations in generator configurations, uncertainties in modeling parameters, and high integration of renewable energy with the grid as an inverter-based resource (IBRs). A scenario is presented where one of the generator's parameters is varied and some of the generators are replaced with IBRs, and the resulting impact on the system's dynamic state estimation behavior is investigated. This analysis aims to demonstrate the capability of the proposed DSE framework in capturing the transient behavior of the internal states of the generator regardless of the changes in grid configuration, the model parameters, and the generation mix of future grids with high integration of IBRs. Specifically, the focus is placed on the inertia of the machine, which plays a significant role in grid stability and resiliency, synchronization, transient response, and frequency response, among other factors. Considering Dynamic Situation # 1, the inertia constant of Machine 9 is varied from 34.5 to 0.345 and IBRs are replaced synchronous machines (#3-#7 & #12-#14) in the grid to provide almost 60% of required power to the grid instead of traditional synchronous generators. The corresponding effects on the internal states of the machine are depicted in Fig. 13. From the plots, it can be inferred that the change in inertia has indeed impacted the transient states of the machine. Nevertheless, despite the modified parameter in this specific case (referred to as Case B in Figure 13), the PF was initially designed using original dynamic data for this scenario, the dynamic estimator still has the capability of tracking the internal states of the synchronous machine successfully. Surprisingly, the tracking and estimation accuracy of PF remains unaffected by the altered parameter. Tracking of each state with nominal parameters is labeled as Case A in the simulations. It can be seen that the developed DSE method is robust and resilient to the change in the topology of the grid, change in the power sources either IBRs or synchronous machines, and changes in the parameters of the synchronous machines where the PF-based DSE is applied directly. Although the integration of IBRs and the changes in parameters of the synchronous machine under investigation might change the dynamic behavior (Cases A and B), nevertheless, the proposed PF-based DSE can successfully track the internal dynamic states of the machines in both cases as seen in Fig. 13. This confirms the capabilities and the resilience and robustness of the proposed method and its applicability for providing another degree of resiliency to the grid through robust monitoring that can be trusted for proper corrective and protective actions in case of events in future grids with high integration of IBRs. In this section, numerical simulations demonstrate the effectiveness of using a set of particles to capture the non-parametric representation of the posterior distribution, particularly in scenarios involving non-Gaussian distributions. Theoretically, the following section presents a performance measure to evaluate the proposed estimation framework.

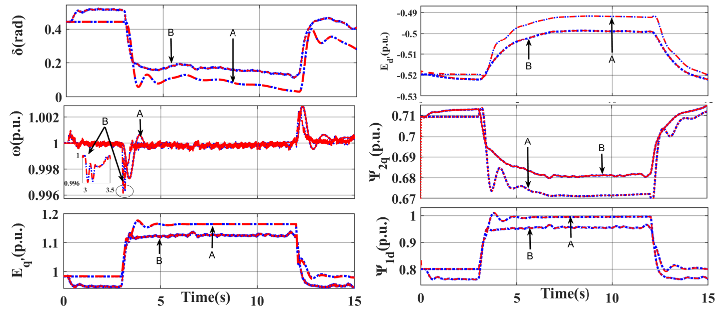

Fig. 13. Simulation results of Dynamic Scenario #1 for M/c 9: Effect of changing parameter on the States estimation.

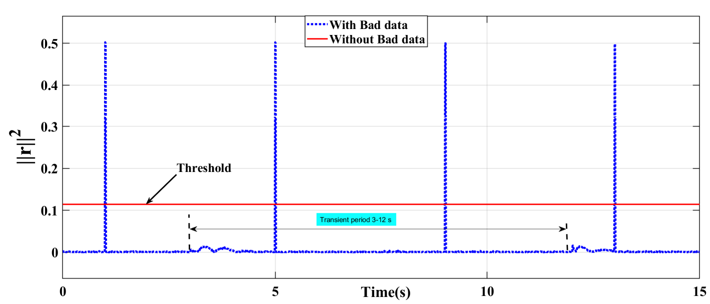

Fig. 14. Case A: Simulation results of Dynamic Scenario #1 for M/c 9: Effect of bad data on output error dynamics.

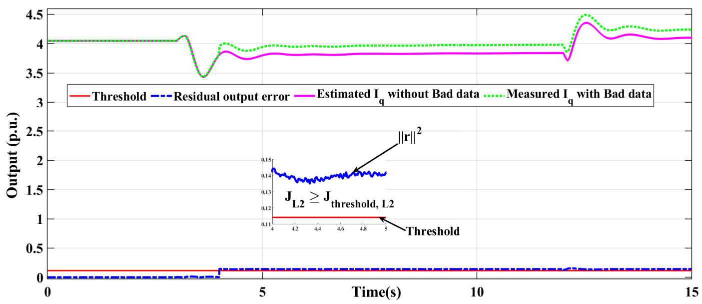

Fig. 15. Case B:Simulation results of Dynamic Scenario #1 for M/c 9: Effect of bad data on output error dynamics.

C. Bad data detection

This section outlines the procedure for detecting local bad data in the i^{th} machine based on its model, which involves computing the estimate of the local state x_i and the local output i_g. Once these estimates are obtained, they are compared to the actual measurements acquired from sensors or measurement devices. The objective of this comparison is to identify any significant discrepancies between the estimated values and the actual measurements. If the differences between the estimated values and the actual measurements surpass certain predetermined thresholds, the data for the i^{th} machine is classified as "local bad data." This indicates that the measurements obtained from this specific machine are likely to be erroneous or corrupted. Identifying and flagging such bad data is crucial to ensure the accuracy, reliability, and resiliency of the overall system's state estimation. Properly handling and filtering out these erroneous measurements can help improve the integrity of the data used for subsequent analysis and decision-making processes.
The identification of bad data follows the expression:

$$||r||^2 = (z_i - \widehat{z}_i)^2 \leq J^2 \tag{20}$$

where z_i and $\widehat{z}_i$ denote the actual output measurement (from the sensor) and estimated output respectively provided by the PF-base DSE approach. J is the threshold defined here to indicate the presence of bad data in the measurement set. This threshold is used for residual evaluation that extracts the feature from the residual signal to detect the outlier. Norm of signal such as L_2 norm (related to RMSD) which signifies as energy of signal can be used to quantify the expression (20). We can formulate the evaluation function $J_{threshold,L_2}$ as

$$J_{threshold,L_2} = \sup(\text{Faultfree})||r||^2 \tag{21}$$

Then, the criterion to detect any bad data or outlier becomes

$$\begin{aligned} J_{L_2} \geq J_{threshold,L_2} \quad &\text{Alarm, bad data is present} \\ J_{L_2} \leq J_{threshold,L_2} \quad &\text{No Alarm, No bad data} \end{aligned} \tag{22}$$

In a particle filter-based estimation process, particles represent possible states of the generator. The prediction step uses a mathematical model to forecast the generator's next state. By comparing the particles with actual measurements, weights are assigned to them. If a fault occurs, inconsistencies between predicted and measured values emerge, causing weights of fault-consistent particles to decrease over time. A threshold is set to decide the fault presence. When fault-consistent particle weights fall below the threshold, the filter raises an alarm, enabling real-time fault detection and timely corrective actions. The efficacy of PF against the detection of bad data is demonstrated using two cases: Case A: Sequential entry of high spike bad data in the measurements.
Case B: Continuous persistence bad data in the measurements. Both cases are demonstrated in Fig. 14 and Fig. 15 that illustrate the effect of bad data incursion in the current measurements of I_q in terms of RMS and norm of the residual signal. Further, the bad-data alarm is indicated with the help of logic mentioned in 22.
It is worth mentioning that a comprehensive analysis is conducted to compute the Mean Square Deviation (MSD) between actual and estimated values of each machine's state under a variety of transient conditions, diverse operational scenarios, and varying levels of measurement errors. Through extensive simulations, the maximum MSD as a threshold is determined that could be employed without causing unwarranted false alarms. This tailored approach effectively addressed the specific requirements of our application, ensuring a robust and dependable bad data detection process.

D. Analytical performance evaluation through CRLB of Estimation

The Computation CRLB serves as a metric for evaluating the performance of an estimator, assuming that the probability distribution of noise (which may be Gaussian or non-Gaussian) corrupting both the state evolution and observation is known. In the state estimation $\widehat{x}$ process, the uncertainty in the system causes estimation error, e_t. These estimation errors can be expressed as $e_t = [\widehat{x} - x]$. The significance of the following inequality lies in its indication that the expectation, $\mathbb{E}$, of the estimation error at any time, t, should be greater than or equal to the inverse of the Fisher Information Matrix, J:

$$\mathbb{E}\left(e_t e_t^T\right) \geq J^{-1} \tag{23}$$

where, Fisher Information matrix, J is expressed as:

$$\begin{aligned} J &= \mathbb{E}\left(-\frac{\partial^2}{\partial e^2}\log p(e_t)\right) \\ J &= \mathbb{E}\left(-\frac{\partial^2}{\partial e^2}\log(\frac{1}{2s_e}\exp\left(-\frac{|e-\theta_e|}{s_e}\right)\right) \end{aligned} \tag{24}$$

In this particular study, the assumption of the error PDF following the same distribution as the state evolution due to noise i.e. i.e. $p(x|y) = p(e_t)$. This is modified to accommodate non-Gaussian distributions, specifically Laplacian noise. Referring to Equation (19), the expression of error likelihood function can be written as follows:

$$\begin{aligned} L(\theta_e, s_e) = \sum_{j=1}^{n} f(x,\theta,s) &= \sum_{j=1}^{n} \frac{1}{2s_e}\exp\left(-\frac{|e-\theta_e|}{s_e}\right) \\ &= \frac{1}{(2s_e)^n}\exp\left(-\frac{\sum_{j=1}^{n}|e-\theta_e|}{s_e}\right) \end{aligned} \tag{25}$$

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In the context of error evolution, s_e and θ denote the same terms as defined in equation (19) for the state. Computation of log likelihood function of (25):

$$\log L(\theta_e, s_e) = \log \frac{1}{2s_e} + \sum_{j=1}^{n} \left(-\frac{|e-\theta_e|}{s_e} \right) \tag{26}$$

Computation of first partial derivative of 26:

$$\begin{aligned} \frac{\partial}{\partial e} \log L(\theta_e, s_e) &= -\frac{1}{s_e} \sum_{n=0}^{n-1} (e[n] - \theta_s) \\ &= -\frac{n}{s_e} (\bar{e} - \theta_e) \end{aligned} \tag{27}$$

Again partial derivative of (27):

$$\begin{aligned} \frac{\partial^2}{\partial e^2} \log P(e) &= -\frac{n}{s_e} \\ J^{-1} &\approx -\frac{s_e}{n} \end{aligned} \tag{28}$$

Given the assumption $\mathbb{E}(ee^T) = J^{-1}$ and referring to equation 28, it can be deduced that with increasing iterations, n, the inverse Fisher information matrix converges to a bounded scalar value (very small). This indicates the convergence of the estimation error covariance, demonstrating the effectiveness of the proposed PF as an efficient estimator even when dealing with non-Gaussian noise distributions.

VIII. Performance Analysis

The estimation process is simulated considering different transients on a multi-machine test power systems. The following are the main findings from the simulation results.

- The simulation results show that the suggested estimation approach has excellent tracking capacity during DSE, even when the system is subjected to various transients and uncertainties. The outcome also includes the RMSD of all states in all scenarios, demonstrating the impact of noise level on the state estimates.
- The computational feasibility of the proposed algorithm is validated by observing the iteration time of the estimator which is found to be less than each sampling time of the measurement. This ensures the real-time application of the proposed DSE. In addition, the proposed DSE algorithm is robust to bear with the changing sampling rate of the system indicating its suitability for practical implementation.
- The PF estimation methodology used in [9] is applied to a less complex model with limited transient scenarios compared to present research, and it is validated that the proposed algorithm is more effective irrespective of the system size. The estimates obtained using PF in our case have the same range of RMSD (of various states) as the reported work (refer Table 1 of [9] and Table II of this paper. Further, in the proposed work, the state variables of a synchronous generator with different types of excitation is considered
- During each case study, the average computation time is smaller than the PMU scan time, resulting in good tracking outcomes. The accuracy-implementation complexity trade-off is investigated using PF and UKF and the results obtained during the simulation study support the feasibility of using particle filters for DSE in complex and nonlinear systems.
- It is observed that the proposed estimation technique is robust and has good tracking capabilities with small estimation errors even though the covariances of the noises increase. Likewise, in case PF, a very minor change in RMSD is noticed during noise level increases compared to UKF, for example, the RMSD for state 1 (δ_{13}) with $100 *$ Noise is 11.23×10^{-3} for PF and 11.72×10^{-3} for UKF for Scenario #1 of M/c 13. Similarly, when comparing PF to UKF, a very minimal change in RMSD is observed as the noise level increases. For example, the RMSD for state 1 (δ_{13}) with $100 *$ Noise is 11.23×10^{-3} for PF and 11.72×10^{-3} for UKF for scenario #1 of M/c 13. As a result, the suggested PF algorithm is robust even when the noise level is increased hundreds of times their base-case values.
- As compared to UKF, implementation of PF algorithm do not require the initial estimates of the noise covariance. Thus, even in the absence of exact knowledge of PMU measurement accuracy, PF itself provides best estimation due to its inbuilt capability of algorithm. This algorithm approximates the PDF using samples drawn from the corresponding distribution called particles using sequential importance sampling and resampling techniques. As a result, noise distributions have no effect on the PF algorithm. This result supports the implementation feasibility of the proposed PF as a state estimator in dynamic and complex system models.
- It is observed from the simulation results in Fig. 11 and Fig.12 that the proposed PF is not only capable to track the state variables under Gaussianity assumption of the system process or measurement noises but also in the presence of non-Gaussian noise. When the process and measurement noise are non-Gaussian, the estimated states likewise have a non-Gaussian distribution. UKF sigma points fail to capture these statistics and deviate. In contrast, PF is unaffected by noise statistics and offers better performance of estimation. The reason for the robustness of PF in the presence of any arbitrary noise distribution is that the filtering process approximates the density rather than the models. This confirms the capabilities and robustness of the proposed method and provides the grid with resilience in terms of such technical issues.
- The extensive study investigates that the execution time of the proposed algorithm depends upon the type of resampling approach and number of particles chosen. Although in the present work, both of these parameters have little effect on estimating accuracy (in terms of RMSD).
- Table IV summarises a comparative study between the published works in literature and proposed work of PF based DSE applications in synchronous machines with regard to multi-machine power systems. The simulation case studies in [18], [9], [16] and in the proposed work are more or less similar, and the performance of an estimator is obviously dependent on numerous aspects (such as transient type and duration, noise distribution, processor speed, and many more) as stated in the literature study. However, an inference can be drawn from the comparative analysis that

TABLE I
EFFECT OF LEVEL OF NOISE ON THE ESTIMATION OF STATE 1 AND STATE 2 OF DIFFERENT MACHINES.

Scenerio	Machine / Error →	M/c 3 δ	M/c 3 ω	M/c 9 δ	M/c 9 ω	M/c 13 δ	M/c 13 ω
1	Noise	1.16	0.26	2.13	1.41	11.1	1.15
	100× Noise	3.37	0.26	3.86	1.41	11.48	1.15
	1000× Noise	10.06	0.26	10.22	1.41	15.02	1.15
2	Noise	1.04	0.23	1.98	1.37	0.72	0.03
	100× Noise	2.97	0.23	3.67	1.37	3.24	0.03
	1000× Noise	9.96	0.23	10.17	1.37	9.9	0.03
3	Noise	2.29	0.51	4.2	2.9	22.1	0.92
	100× Noise	5.92	0.51	5.8	2.9	22.3	0.92
	1000× Noise	17.2	0.51	0.52	15.82	24.4	0.92

*Range of RMSD: For δ in 10^{-3} and For ω in 10−5.

TABLE II
COMPARATIVE ANALYSIS OF RMSD OF PF AND UKF FOR 6 STATES FOR M/C 13

Dynamic Situation→	#1		#2		#3	
State Estimator →	PF	UKF	PF	UKF	PF	UKF
δ_{13}	11.1	11.60	0.72	9.56	22.1	956.2
ω_{13}	1.15	517	.03	4.36	0.92	41.36
$E_{q'13}$	7.3	20.7	6.63	17.18	6.61	17.18
$E_{d'13}$	15.74	15.71	12.32	12.29	12.45	12.29
$\psi_{2q_{13}}$	60.32	60.32	58.6	58.6	58.63	58.6
$\psi_{1d_{13}}$	24.3	26.89	23.53	25.15	23.52	25.15

*Range of RMSD: For δ in 10^{-3}; For $\omega, E_{q'}, E_{d'}, \psi_{2q}, \psi_{1d}$ in 10^{-5}

the proposed PF is robust and it has the potential to address the issues of DSE for complex, stochastic (with variable noise structure) systems with the feasibility of practical implementation as well, especially for 7^{th} dynamic model of synchronous machines. The developed fully decentralised PF-based DSE method in this paper paves the way for developing the next generation of energy management systems and novel system monitoring, control and protection tools to achieve better reliability and resiliency of future power systems.

TABLE III
COMPARISON OF EXECUTION TIME(MS) FOR SCENARIO #1 FOR M/C 9 WITH DIFFERENT PF RESAMPLING METHODS

No. of Particles →	1		100		1000	
Noise →	GN	LN	GN	LN	GN	LN
Systematic	0.926	1.03	1.36	1.36	2.33	2.4
Multinomial	0.9202	1.027	0.93	1.26	1.78	2.67
Stratified	0.93	1.04	0.96	1.26	1.86	2.1

GN-Gaussian Noise; LN-Laplacian Noise

IX. CONCLUSION

This paper investigated the feasibility of applying particle filters to modern power systems as a dynamic state estimator for enabling the applications of DSE in power system control and protection applications to improve the resiliency and security of the grid based on advanced and trusted monitoring functionalities. The proposed PF-based DSE technique was tested on a multi-machine system considering various forms of transients, observation and innovation outliers, and fault situations. The paper developed an approach for bad-data detection which can help in monitoring such issues and mitigate their impacts on grid operation security and resiliency. The simulation study validated the superior performance of the proposed filter algorithm with respect to noise level, transient scenarios, and system configuration. Furthermore, the suggested DSE technique can track states even with typical noises with strong tail distributions, which are more difficult to deal with than Gaussian noise. The findings suggest that combining PF-based DSE with upcoming phasor measuring technologies is a promising strategy for improving interconnected power system command and control. Furthermore, the computational efficiency of the proposed PF is compared to that of the UKF for the same multi-machine model in terms of estimation accuracy and implementation efforts, and the findings support the feasibility of utilising particle filters for DSE. Thus, the decentralised estimating approach using PF has confirmed its viability and performance improvement on a benchmark system. The study's findings suggest that PF-based dynamic state estimation can be used with forthcoming phasor measuring technologies in the future. Finally, the proposed decentralised particle filtering technique can offer many potential applications of DSE in the operation and control of the power system to improve the overall security and resiliency of the grid based on proper corrective and protective actions. The results presented in the paper confirmed the proposed method's online applicability and the comparative study confirmed the proposed method's superiority. As future work, the proposed method would be developed for application in power system control to make power system controllers more resilient against cyberattacks, and faulty and noisy measurements to improve the overall resiliency and reliability of the grid in case of severe events. Furthermore, other types of stochastic and deterministic DSE methods can be investigated for 7th dynamic order model of synchronous machines. As future work, an adaptive threshold will be developed for detecting bad data and a residual and threshold will be developed for detecting malware such as cyberattacks.

ACKNOWLEDGMENT

The work of H.H. Alhelou is supported by The Commonwealth Scientific and Industrial Research Organisation (CSIRO) under the Global Power System Transformation Consortium (G-PST) program and the Monash-Woodside partnership project. The authors Pierluigi Siano and Mohammed AL-Numay acknowledge financial support from the Researchers Supporting Project Number (RSP2023R150), King Saud University, Riyadh, Saudi Arabia.

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TABLE IV
COMPARISON BETWEEN REPORTED AND PROPOSED WORK: PF-BASED DSE OF SYNCHRONOUS MACHINE WITH MULTI-MACHINE SETUP

	[18]	[9]	[16]	**Proposed Work**	
Dynamic Order & Model	4th (IEEE 39 Bus)	6th (IEEE 14 Bus)	4th (IEEE 39 Bus)	7th (IEEE 68 Bus)	7th (IEEE 68 Bus)
Noise Considered	GN($(10^{-8} - 10^{-20}$)	GN(10^{-4})	GN($10^{-6} - 10^{-8}$) + LN ($\mu = 0, s = 0.15$)	GN($10^{-5} - 10^{-8}$) Not tracked with LN	GN($10^{-6} - 10^{-8}$) + LN ($\mu = 0, s = 1/\sqrt{2}$)
Estimator	PF with SS	PF with SS	Intelligent PF with IS	UKF	PF with SS, MS, and StS
T_{sim}(s)	15	20	25	15	15
Av. T_c (ms)	6.96 -12.8 (with 400 -1200 nop)	Not mentioned but kept Sample time of DSE= 5ms	12.1	1.5	$0.92 > T_c < 2.9$
T_s (ms)	20, 30	33.33	20	10	10
Range of Error (δ)	Not Mentioned	0.02-0.03(RMSD) (with 50-150 nop)	$0.42 \times 10^{-4} - 12 \times 10^{-4}$ (CEI)	$> 1.5 \times 10^{-3} - \infty$ (RMSD) (Not tracked for some M/cs and Transients.)	$1 \times 10^{-3} - 12 \times 10^{-3}$ (RMSD) with 1-1000 nop

GN(.)- Gaussian Noise(Covariance (Q_w, ν)); LN-Laplacian Noise; T_c ; Computation Time; T_{sim}-Simulation Time; T_s-Sample Time; SS-Systematic Sampling; MS-Multinomial Sampling; StS-Starified Sampling; nop-No. of Particles; CEI- Cumulative error index

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